**Beta & Gamma Functions**

**Beta Function or First Eulerian Integral:** A function of the form,



is called Beta function or first Eulerian integral and it is denoted by, .

.

**Gamma Function or Second Eulerian Integral:** A function of the form,



is called Gamma function or second Eulerian integral and it is denoted by, .

.

**Properties of Beta and Gamma functions:**The properties are given below:

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. .
9. **.**

**Theorem-01:** Prove that .

**Proof:** We know that the beta function is



Let 





From (1) we get,





 **(Proved)**

**Theorem-02:** Prove that .

**Proof:** We know that the beta function is



If then,

































 **(Proved).**

**Theorem-03:** Prove that i). ; ii). ; iii). 

**Proof:** We know that the Gamma function is



If then, from (1) we get,













  **(Proved)**

Again, replacingbyin (1) we get,



 [Integrating by parts]





 **(Proved)**

Again, from (1) we get,



 [Integrating by parts]





 **(Proved)**

**Theorem-04:** Prove that i). ; ii).  for  is a +ve integer.

**Proof:** If  is a positive integer then,



















 **(Proved)**

Again if  is a positive integer then,



















 **(Proved)**

**Theorem-05:** Prove that .

**Proof:** We know that the beta function is



Let 





From (1) we get,







  **(Proved)**

**Ex-01:** Prove that .

**Theorem-06:** Establish the relation between Gamma and Beta function.

**Or,** Prove that .

***Proof:*** From the definition of Gamma function we can write



Assume that .

Limit: when, then and when, then.

From above relation we have







Again,



Multiplying (i) and (ii) we get















 **(Proved)**

**Theorem-07:** Prove that ****.

**OR**

**Evaluate**, in terms of the Gamma function, the integral ****

**Proof:** We know that

Let .

Limit: .

Now,







 .

Now from above equation we get



Using the relation between beta and gamma function**,** we have



 **(Proved)**

**Theorem-08:** Prove that ****, where ****.

**OR**

Establish Euler’s reflection formula.

**Proof:** We know that



and 

Now from (1) and (2), we have



Putting  and so  in (3), we get





Again we know that the formula from integral calculus,



The equation (4) can be written as,



Now replacing m by n we have

 **(Proved)**

<https://mathoverflow.net/questions/76399/one-line-proof-of-the-eulers-reflection-formula>

**Problem-01:** Evaluate  **Exer.-01:**

**Solution:** Let, **Ans:**

 **Exer.-02:**

 **Ans:**

 **Exer.-03:**

 **Ans:**







 **(Ans.)**

**Problem-02:** Evaluate 

**Solution:** Let,















 **(Ans.)**

**Problem-03:** Evaluate  **Exer.-04:**

**Solution:** Let, **Ans:**

 **Exer.-05:**

 **Ans:**

 **Exer.-06:**

 **Ans:**

 **(Ans.)**

**Problem-04:** Evaluate  **Exer.-04:**

**Solution:** Let, **Ans:**

 **Exer.-05:**

 **Ans:**

















 **(Ans.)**

**Problem-05:** Evaluate  **Exer.-06:**

**Solution:** Let, **Ans:**

















 **(Ans.)**

**Problem-06:** Evaluate  **Exer.-07:**

**Solution:** Let, **Ans:**

 **Exer.-08:**

 **Ans:**























 **(Ans.)**

**Problem-08:** Evaluate  **Exer.-09:**

**Solution:** Let, **Ans:**

Put  **Exer.-10:**

 **Ans:**

Now,  **Exer.-11:**

 **Ans:**













 **(Ans.)**

**Problem-09:** Evaluate  **Exer.-12:**

**Solution:** Let, **Ans:**

Put 



Now, 















 **(Ans.)**

**Problem-10:** Evaluate 

**Solution:** Let,

Put 



Now, 



















 **(Ans.)**

**Problem-11:** Evaluate  **Exer.-13:**

**Solution:** Let, **Ans:**











 **(Ans.)**

**Problem-12:** Show that **Exer.-14: Show that**

**Solution:** Let, **Exer.-15: Show that** 

Put, 





Now,









 **(Showed.)**

**Problem-13:** Show that **Exer.-16: Show that** 

**Solution:** Let,











 **(Showed)**

**Problem-14:** Show that **Exer.-17: Show that** 

**Solution:** Let,











 **(Showed.)**